MA 362 Quiz 8 – Solvable Galois groups – Friday, April 19

1. Let F be a field and consider the polynomial $f(x) = x^n - a \in F[x]$. Let K be the splitting field of $f(x)$. What does lemma 56.3 say about this situation? (choose one)

   a. $f(x)$ is solvable by radicals.

   b. $G(K/F)$ is a solvable group.

   c. If $\text{char}(F) = 0$ then $G(K/F)$ is a solvable group.

   d. If F contains all the roots of $x^n - 1$, then $G(K/F)$ is a solvable group.

2. Let $F \subseteq E$ be a finite normal extension of fields with characteristic 0. What is the content of Theorem 56.4 in this situation? (choose one)

   a. If E is contained in an extension of F by radicals, then $G(E/F)$ is sovable.

   b. if $G(E/F)$ is solvable, then E is contained in an extension of F by radicals.

3. For which of these polynomials in $\mathbb{Q}[x]$ does the splitting field have solvable Galois group? (Each answer should be chosen exactly once)

   a. $g(x) = x^5 - 13$

solvable by lemma 56.3.	solvable by theorem 56.4.	not covered by either result.

   b. The splitting field of $f(x) = x^5 - 3x + 1$ is

solvable by lemma 56.3.	solvable by theorem 56.4.	not covered by either result.

   c. $h(x) = x^8 - 14x^4 + 9$, a polynomial whose eight roots are $x = \pm\sqrt{\pm\sqrt{2} \pm \sqrt{5}}$

solvable by lemma 56.3.	solvable by theorem 56.4.	not covered by either result.