MA 362 — Homework 12 — Due Friday, April 26

Final Exam Review

The material that we've covered this semester is (properly) contained in sections 29, 30, 31, 48, 49, 50, 51, 53, 54, 35 and 56 in Fraleigh. It's not possible for a five-question homework assignment to be an adequate review for all this material, so you shouldn't think of these questions as comprehensive for the final exam. The exercises and "concepts" questions at the ends of each section are good places to look for additional practice problems.

1. Prove or disprove

   a. If $F \subseteq E$ is a field extension and $\alpha \in E$ is algebraic over F, then any irreducible polynomial $f(x)$ with α as a root is divisible by the minimal polynomial of α over F.

   b. If $F \subseteq E$ is an algebraic extension, then it is a simple extension.

   c. If $F \subseteq E$ is a simple extension, then it is an algebraic extension.

   d. If $F \subseteq E$ is a field extension and $y \in E$ is transcendental over F, then y^2 is transcendental over F.

   e. If $F \subseteq E \subseteq K$ are fields and $\alpha \in K$ is algebraic over F, then α is also algebraic over E.

   f. If $F \subseteq E \subseteq K$ are fields and $\alpha \in K$ is transcendental over F, then y is also transcendental over E.

2. Let $F \subseteq E$ be a field extension. Prove that:

$$|G(E/F)| \leqslant \{E : F\} \leqslant [E : F].$$

3. Let $F \subseteq E$ be a finite degree field extension. Prove that if the fixed field of $G(E/F)$ is F, then E is a finite normal extension of F.

4. Using results that were proven earlier in class, prove the Main Theorem of Galois theory.

5. Prove that for *any* $n \geqslant 5$, there exist polynomials of degree n that are not solvable by radicals. You may use the facts that (1) A_n is a normal, index 2 subgroup of S_n for all n, and that (2) is a subgroup $G \leq S_n$ contains a two-cycle and an n-cycle, then $G = S_n$.

Bonus. Go over exams 1 and 2 and re-do any problems where you lost a significant number of points.