

# Solutions

## MA 362 Quiz 8 – Solvable Galois groups – Friday, April 19

1. Let  $F$  be a field and consider the polynomial  $f(x) = x^n - a \in F[x]$ . Let  $K$  be the splitting field of  $f(x)$ . What does lemma 56.3 say about this situation? (choose one)

☐ a.  $f(x)$  is solvable by radicals.

☐ b.  $G(K/F)$  is a solvable group.

☒ c. If  $\text{char}(F) = 0$  then  $G(K/F)$  is a solvable group.

☐ d. If  $F$  contains all the roots of  $x^n - 1$ , then  $G(K/F)$  is a solvable group.

2. Let  $F \subseteq E$  be a finite normal extension of fields with characteristic 0. What is the content of Theorem 56.4 in this situation? (choose one)

☒ a. If  $E$  is contained in an extension of  $F$  by radicals, then  $G(E/F)$  is solvable.

☐ b. if  $G(E/F)$  is solvable, then  $E$  is contained in an extension of  $F$  by radicals.

3. For which of these polynomials in  $\mathbb{Q}[x]$  does the splitting field have solvable Galois group? (Each answer should be chosen exactly once)

a.  $g(x) = x^5 - 13$

☒ solvable by lemma 56.3.

☐ solvable by theorem 56.4.

☐ not covered by either result.

b. The splitting field of  $f(x) = x^5 - 3x + 1$  is

☐ solvable by lemma 56.3.

☐ solvable by theorem 56.4.

☒ not covered by either result.

c.  $h(x) = x^8 - 14x^4 + 9$ , a polynomial whose eight roots are  $x = \pm\sqrt{\pm\sqrt{2} \pm \sqrt{5}}$

☐ solvable by lemma 56.3.

☒ solvable by theorem 56.4.

☐ not covered by either result.