

MA 362 Quiz 3 – Extensions of finite degree – Wednesday, Feb 6

1. Let F be a field and consider a field extension $F(\alpha, \beta)$ where $[F(\alpha) : F] = 3$ and $[F(\alpha, \beta) : F(\alpha)] = 2$

(a) What is the degree $[F(\alpha, \beta) : F]$?

(b) Use α and β to write down a basis for $F(\alpha, \beta)$ as an F -vector space.

2. Let $F \subseteq E \subseteq K \subseteq L$ be fields, and suppose that $[L : F] = 42$, $[E : F] = 2$, and $[L : K] = 3$. What is $[K : E]$?

3. Suppose $F \subseteq E$ is a field extension and $[E : F]$ is finite. Explain why E is an *algebraic* extension of F . (You don't need to do a whole careful proof. Just explaining the key ideas is enough.)