

MA 362 Quiz 2 – Concepts – Wednesday, Jan 30

1. (2 pts) Consider the real number  $\alpha = \frac{\sqrt{2}}{2}$ . Find the *minimal polynomial* of  $\alpha$  over  $\mathbb{Q}$ .

**Reminder:** the minimal polynomial has to be both *irreducible* and *monic*.

- 2a. (3 pts) Let  $F$  be a field and let  $E = F(\alpha)$  be a simple extension where  $\alpha$  is algebraic over  $F$ , and its minimal polynomial has degree 4. Use  $\alpha$  to write down a basis for  $E$  as an  $F$ -vector space.

- b. (2 pts) What is  $[E : F]$ ? (Your answer should be a number.)

(More on other side.)

3. (3 pts) Consider the field extension  $\mathbb{Q} \subseteq E = \mathbb{Q}[x]/(x^3 + x + 1) \in E$ . Let  $\alpha$  be the element  $x + (x^3 + x + 1)$ . In other words,  $\alpha$  is the coset in the quotient ring represented by  $x$ . Write  $\alpha^4$  as a linear combination (with  $\mathbb{Q}$  coefficients) of  $\alpha^2, \alpha$  and 1.