MA 362 Quiz 5 – Splitting Fields, Separable Extensions – Wednesday, March 6

1. The roots of $x^3 - 5 \in \mathbb{Q}[x]$ are

$$\alpha = \sqrt[3]{5}, \qquad \beta = \sqrt[3]{5}\left(\frac{-1+i\sqrt{3}}{2}\right), \qquad \gamma = \sqrt[3]{5}\left(\frac{-1-i\sqrt{3}}{2}\right).$$

With this information, explain why $E = \mathbb{Q}(\sqrt[3]{5})$ is *not* a splitting field.

x^3-2 has 2 complex roots. Since $E \subseteq \mathbb{R}$, it contains only one of the three roots of x^3-2. So E is not a splitting field.

2. By any method you want, prove that $E = \mathbb{Q}(\sqrt[3]{5})$ is a *separable* extension of $\mathbb{Q}$.

$\mathbb{Q}$ is a perfect field, so any finite extension is separable over $\mathbb{Q}$.

3. For $E = \mathbb{Q}(\sqrt[3]{5})$, find the values below. No justification is necessary:

i. The degree $[E : F] =$ 3

ii. The index $\{E : F\} =$ 3 $\leftarrow$ by defn. of sep.

iii. The size of the automorphism group $|G(E/F)| =$ 1

(any aut sends α to α or β or γ. but only $\alpha \mapsto \alpha$ gives an aut).