$$
\begin{tikzcd}
 & \boxed{\begin{array}{c} G \\ \text{fixes } \mathbb{Q}. \end{array}} \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\
\boxed{\begin{array}{c} \{\sigma_{id}, \sigma_1\} \\ \text{fixes } \mathbb{Q}(\sqrt{3}) \end{array}} \arrow[dr, dash] & \boxed{\begin{array}{c} \{\sigma_{id}, \sigma_2\} \\ \text{fixes } \mathbb{Q}(\sqrt{2}) \end{array}} \arrow[d, dash] & \boxed{\begin{array}{c} \{\sigma_{id}, \sigma_3\} \\ \text{fixes } \mathbb{Q}(\sqrt{6}) \end{array}} \arrow[dl, dash] \\
 & \boxed{\begin{array}{c} \{\sigma_{id}\} \\ \text{fixes } \mathbb{Q}(\sqrt{2}, \sqrt{3}) \end{array}} &
\end{tikzcd}
$$

3. $x^5 - 1 = (x^4 + x^3 + x^2 + x + 1)(x-1)$, and

i) the roots of this factor are

$w = e^{\frac{2\pi i}{5}}$, $w^2 = e^{\frac{4\pi i}{5}}$, $w^3 = e^{\frac{6\pi i}{5}}$, $w^4 = {}^{\frac{8\pi i}{5}}$.

The splitting field of $x^5 - 1$ is the smallest subfield of $\mathbb{Q}$ that contains all of these roots, which is $\mathbb{Q}(w)$.

ii) $[E : \mathbb{Q}]$ = deg of <u>min</u> poly of $w = 4$

iii) $\mathbb{Q}$ ~~separable~~ perfect $\Rightarrow$ E separable over $\mathbb{Q}$ $\Rightarrow$
$\{E : \mathbb{Q}\} = [E : \mathbb{Q}] = 4$.

iv. $|G(\mathbb{Q}(w)/\mathbb{Q})| = \{\mathbb{Q}(w) : \mathbb{Q}\} = 4,$ ★

and $\sigma \in G(\mathbb{Q}(w)/\mathbb{Q})$ is determined by

$\sigma(w)$ [see problem # 1].

$\sigma(w) \in \{w, w^2, w^3, w^4\}$, since it

can only be a conjugate of w.

Let σ_1 be defined by $w \longmapsto w^1$

σ_2 by $w \longmapsto w^2$

σ_3 by $w \longmapsto w^3$

σ_4 by $w \longmapsto w^4$.

then the mult. table is.

	σ_1	σ_2	σ_3	σ_4
σ_1	σ_1	σ_2	σ_3	σ_4
σ_2	σ_2	σ_4	σ_1	σ_3
σ_3	σ_3	σ_1	σ_4	σ_2
σ_4	σ_4	σ_3	σ_2	σ_1