

MA 362 Quiz 4 - Definitions - Monday, Feb 25

1. The hypotheses of the *isomorphism extension theorem* are given below. What is the conclusion of the theorem?

Theorem: Let $F_1 \subseteq E$ be an algebraic field extension, let $\varphi : F_1 \rightarrow F_2$ be an isomorphism of fields, and suppose $\overline{F_2}$ is the algebraic closure of F_2

there exists an isomorphism from E to a subfield of $\overline{F_2}$ that agrees with φ on elements of F_1

2. Complete the following definitions:

- i. Let F be a field with algebraic closure \overline{F} , and let $F \subseteq E \subseteq \overline{F}$ be an extension of F . We say that E is a *splitting field* if:

there exists a set $X = \{f_i(x) \mid i \in I\} \subseteq F[x]$ such that E is the smallest field contained in \overline{F} that contains all roots of all polynomials in X .

- ii. Let $F \subseteq E$ be a field extension of finite degree. The *index* of the extension, denoted $\{E : F\}$, is:

the number of extensions of ("ways" ^{or} to extend) an isomorphism $F \rightarrow F_2$ to an isomorphism from E to a subfield of $\overline{F_2}$