MA 362 Quiz 5 – Splitting Fields, Separable Extensions – Wednesday, March 6

1. The roots of $x^3 - 5 \in \mathbb{Q}[x]$ are

$$\alpha = \sqrt[3]{5}, \qquad \beta = \sqrt[3]{5}\left(\frac{-1 + i\sqrt{3}}{2}\right), \qquad \gamma = \sqrt[3]{5}\left(\frac{-1 - i\sqrt{3}}{2}\right).$$

   With this information, explain why $E = \mathbb{Q}(\sqrt[3]{5})$ is *not* a splitting field.

2. By any method you want, prove that $E = \mathbb{Q}(\sqrt[3]{5})$ is a *separable* extension of $\mathbb{Q}$.

3. For $E = \mathbb{Q}(\sqrt[3]{5})$, find the values below. No justification is necessary:

   i. The degree $[E : F] =$

   ii. The index $\{E : F\} =$

   iii. The size of the automorphism group $|G(E/F)| =$