MA 362 Quiz 2 – Concepts – Wednesday, Jan 30

1. (2 pts) Consider the real number $\alpha = \frac{\sqrt{2}}{2}$. Find the *minimal polynomial* of α over $\mathbb{Q}$.

Reminder: the minimal polynomial has to be both *irreducible* and *monic*.

$2\alpha = \sqrt{2}$

$4\alpha^2 = 2$

$4\alpha^2 - 2 = 0$

$\alpha^2 - \frac{2}{4} = 0$

so $x^2 - \frac{1}{2}$ is the min. poly of α

2a. (3 pts) Let F be a field and let $E = F(\alpha)$ be a simple extension where α is algebraic over F, and its minimal polynoial has degree 4. Use α to write down a basis for E as an F-vector space.

$\{1, \alpha, \alpha^2, \alpha^3\}$

b. (2 pts) What is $[E : F]$? (Your answer should be a number.)

4 (# of elts in a basis)

(More on other side.)

3. (3 pts) Consider the field extension $\mathbb{Q} \subseteq E = \mathbb{Q}[x]/(x^3+x+1) \in E$. Let α be the element $x+(x^3+x+1)$. In other words, α is the coset in the quotient ring represented by x. Write α^4 as a linear combination (with $\mathbb{Q}$ coefficients) of α^2, α and 1.

we know:

$$\alpha^3 + \alpha + 1 = 0.$$

So:

$$\alpha^4 + \alpha^2 + \alpha = 0$$

$$\Rightarrow \quad \alpha^4 = -\alpha^2 - \alpha.$$