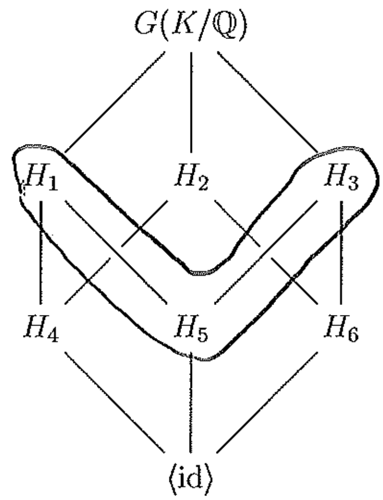
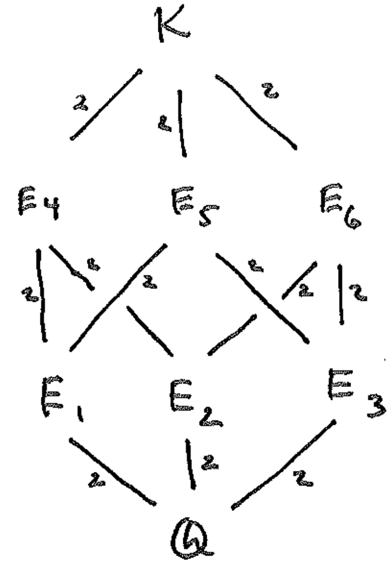
Solutions

MA 362 Quiz 6 – The Main Theorem of Galois Theory – Friday, April 5

Let K be a finite normal extension of $\mathbb{Q}$ with $[K:\mathbb{Q}] = 8$, and suppose that the Galois group $G(K/\mathbb{Q})$ has the subgroup lattice shown below.





For each subgroup H_i, we let E_i be its fixed field.

1. (4 pts) Suppose that $|H_1| = |H_2| = |H_3| = 4$, and that $|H_4| = |H_5| = |H_6| = 2$. Find the following values:

   $[K : E_5] = \boxed{2}$ $\quad [E_3 : \mathbb{Q}] = \boxed{2}$ $\quad |G(K/E_1)| = \boxed{4}$

2. (2 pts) What is the fixed field of $H_1 \cap H_3$? Your answer should be one of the fields in the set $\{\mathbb{Q}, E_1, E_2, E_3, E_4, E_5, E_6, K\}$. $H_1 \cap H_3 = H_5$, which fixes E_5.

3. (2 pts) What is the group of automorphisms fixing $E_5 \cap E_6$? Your answer should be one of the groups in the set $\{\langle \text{id} \rangle, H_1, H_2, H_3, H_4, H_5, H_6, G(K/\mathbb{Q})\}$.

   $E_5 \cap E_6 = E_3$, and $G(K/E_3) = H_3$

4. (2 pts) Of the four field extensions below, circle the ones that are *guaranteed* to be finite normal extensions. any subfield of K

   $\boxed{E_5 \subseteq K}$ $\quad \boxed{E_3 \subseteq E_5}$ $\quad \boxed{E_3 \subseteq K}$ $\quad \boxed{\mathbb{Q} \subseteq E_3}$