

MA 362 Quiz 2 – Concepts – Wednesday, Jan 30

1. (2 pts) Consider the real number $\alpha = \frac{\sqrt{2}}{2}$. Find the *minimal polynomial* of α over \mathbb{Q} .

Reminder: the minimal polynomial has to be both *irreducible* and *monic*.

- 2a. (3 pts) Let F be a field and let $E = F(\alpha)$ be a simple extension where α is algebraic over F , and its minimal polynomial has degree 4. Use α to write down a basis for E as an F -vector space.

- b. (2 pts) What is $[E : F]$? (Your answer should be a number.)

(More on other side.)

3. (3 pts) Consider the field extension $\mathbb{Q} \subseteq E = \mathbb{Q}[x]/(x^3 + x + 1) \in E$. Let α be the element $x + (x^3 + x + 1)$. In other words, α is the coset in the quotient ring represented by x . Write α^4 as a linear combination (with \mathbb{Q} coefficients) of α^2 , α and 1.